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NOTS ANALOG COMPUTING FACILITY

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AVIATION ORDNANCE AND TEST DEPARTMENT

U.S. NAVAL ORDNANCE TEST STATION

NOTS ANALOG COMPUTING FACILITY

By

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PRELIMINARY DATA

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PRELIMINARY DATA

NOTS ANALOG COMPUTING FACILITY

ABSTRACT

The equipment available at NOTS for solution of problems involving differential and algebraic equations by electric analog methods is described in detail. An active program of construction and procurement of additional apparatus to accommodate a greater variety of problems is outlined. Now available are:

(1) 30 stable d-c computer amplifiers of relatively simple design. Any one of these may be used as an integrator, a summing amplifier, a sign changer, or a differentiator.

(2) Six d-c amplifiers driving a six-channel Brush recorder. Each amplifier may be used to record one voltage function or the sum of several as desired.

(3) Six ganged motor-driven potentiometers accommodating coefficients varying linearly or inversely with time.

To be available soon:

(1) Eight to twelve multiplying circuits to solve non-linear equations.

(2) Six input devices for insertion of arbitrary functions to an accuracy of 0.5 per cent.

Design of networks for easy and accurate solution of typical problems is discussed.

INTRODUCTION

This report is written to describe in detail the analog computing equipment available at NOTS and to outline its intended uses so that anyone not previously familiar with this or similar equipment may set up his own problems for solution.

The work has been done under Task Assignment NOTS-32-ReSc-114-1. The early analog computer work done at NOTS was reported in

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NAVORD 1948. Results of an early simulator study of aircraft fire control problems were reported in Technical Memorandum RDAF-19.

DESCRIPTION OF FUNDAMENTAL CIRCUIT

Figure 1 shows a schematic diagram of the basic computer used at NOTS connected as an integrator. Interchanging R_1 and C will produce a differentiator. By replacing C by a resistor, the circuit will operate as a direct-current amplifier having a voltage gain equal to the value of this resistor divided by R_1. In each case the input and output voltages are of opposite polarity. Several input voltages may be accommodated by connecting separate resistors between the grid of V_1 and the respective input voltage functions. Thus a summing amplifier or summing integrator is accomplished.

OPERATION

Operation of the circuit as an integrator may be explained as follows: Assuming that the pentode V_1 (Fig 1) draws no grid current, all of the current required to change the potential across C must flow through R_1. The amplifier loop in this circuit is designed to maintain the grid of V_1 at very nearly ground potential by automatically changing the potential of the cathode of V_2. Thus the charging or discharging current of C is exactly proportional to the input voltage to be integrated because the voltage across R_1 is equal to the input voltage. Neglecting for a moment the effect of R_3, the voltage between the cathode of V_2 and ground is equal to $\frac{V_cG}{1+G}$ where V_c is the voltage across C and G is the gain of the amplifier without negative feedback. The effect of adding the proper value of R_3 to the circuit is to increase G to infinity. However, since G is not entirely constant, R_3 cannot be chosen exactly for all conditions. In practice, the net effect of adding R_3 is to increase G to about ten thousand.

The error in potential of V_1 grid (Fig 1), assuming that this point was adjusted to zero potential at the initial output voltage (boundary condition), is equal to $\Delta V_c\left(1-\frac{G}{1+G}\right)$

where ΔV_c is the change in potential difference across C. In practice, this error can be held to about 0.01 volts for a 100-volt change in V_c. Amplifier drift contributes some additional error which can be eliminated by adjustment immediately before use. If such adjustment is omitted, this error in potential will not exceed 0.03 volts over a period of several weeks. In general, the accuracy of computation will be good if the input function to be integrated is many times greater than this total error of potential since it is simply added to or subtracted from the input function.

A physical analysis of the mechanism of the positive feedback established by R_3 reveals that, in order for the cathode of V_2 to change potential, the voltage between grid and cathode of V_1 must also change slightly. The addition of R_3 makes it possible for the required change in potential to occur between cathode and ground, thus allowing the grid to remain at ground potential. The potentiometer R_5 is included in order that the quiescent conditions may be adjusted so that the grid of V_1 assumes ground potential. The resistor R_3 is chosen so that this point remains at ground potential within the usable limits of output voltage.

STABILITY

When this circuit is used as an integrator it is stable, and there are no particular limitations that must be observed in order that stability be maintained since the capacitor which is part of the integration circuit prevents any instability. When used as a direct current amplifier or summing amplifier, the gain must be kept below one half of the gain without feedback. In order to allow a margin of safety, a voltage gain no greater than 50 should be required of any one of the circuits described here. This means that the feedback resistor (the resistor replacing C in Fig 1) can be no more than 50 times as great as all of the input resistors (R_1 in Fig 1) connected in parallel.

When used as differentiators, these circuits will be unstable unless some precaution is taken. The reason for instability is that at high frequencies the differentiator tends to become a very high gain amplifier. The most logical thing to do is to connect a resistor in series with the input voltage in addition to the differentiator capacitor. This resistor must be one fiftieth of the feedback resistor or greater. The need for the stabilizing resistor is the same requirement as the limitation on the gain of a direct-current amplifier or summing amplifier mentioned above. Obviously the stabilizing resistor destroys the accuracy of computation. However, most practical problems can be solved without the use of differentiators; in fact, a true derivative may be obtained by other means, as will be shown later.

DESCRIPTION OF UNITS

The present NOTS analog computer consists of thirty universal computer amplifiers, six additional amplifiers designed to drive a six-channel Brush recorder, and six ganged motor-driven potentiometers. The thirty universal computer amplifiers are packaged in three units, each unit containing ten amplifiers and attached to a standard 8-3/4 x 19-inch relay rack panel. Each ten-channel unit has its own integral cathode heater supply. The critical heaters are supplied from a resonant circuit a-c voltage regulator. Each unit has its own separate electronically regulated d-c power supply, also mounted on an 8-3/4 x 19-inch panel. Each ten-amplifier unit has two 12-contact Jones plugs mounted on the front panel. One of the Jones plugs brings out the respective amplifier input grids to contacts 1 to 10; contact 11 is a ground connection; and contact 12 is unused. The corresponding amplifier outputs are brought out to contacts 1 to 10 of the second Jones plug. Again contact 11 is ground, contact 12 unused. A test point is brought out to a pin jack on the panel. This test point may be switched to any one of the input grids so that it may be adjusted to zero potential by manipulating the corresponding front panel zero adjustment while observing a vacuum tube voltmeter connected to the test point. Each of the thirty universal computing amplifiers is capable of plus or minus 100-volts output at ten milliamperes, or a permissible load of 10,000 ohms. Figures 2 and 3 are schematic wiring diagrams of the ten-channel computer amplifier unit and its regulated power supply. Figures 4 and 5 are photographs of the ten-amplifier unit. The six-amplifier unit is similar in design and construction to the ten-amplifier units except that it is capable of only plus or minus 20-volts output. All power supplies are self-contained, and specifically designed to drive a six-channel Brush recorder. The feedback resistor for each channel is fixed at 200,000 ohms. Six selector switches provide for three different values of input resistors or external connection of input resistors as desired. Figure 6 is a schematic wiring diagram of this unit.

Figure 8 is a photograph of the six ganged motor-driven potentiometers. Each is a 40-turn helical potentiometer. These are used to accommodate coefficients varying linearly or inversely with time.

EXPANSION

In order to increase the variety of problems which may be solved, more apparatus is being constructed and bought. The following items will be available in the near future.

(1) Six input devices for insertion of arbitrary functions of time or other variable to an accuracy of 0.5 per cent.

(2) Eight to twelve multiplying circuits to handle equations having non-linear terms.

HOW PROBLEMS ARE SET IN

Problems are set into the computer by building a "breadboard" of precision resistors and accurate capacitors and plugging it into the computer amplifiers. It is possible to devise combinations of these computer circuits analogous to any physical problem expressible in terms of linear differential equations or systems of differential equations. Coefficients varying linearly or inversely with time (the only independent variable) can be provided for by the motor-driven potentiometers.

ORDINARY LINEAR EQUATIONS

In order to reduce a differential equation to a form which is easy to handle, it should be differentiated until no integral terms are present, and the resulting equation solved for the highest order derivative. This highest order derivative is assumed to be represented by the voltage output of a computing circuit connected so as to add several functions into one voltage. Enough integrators are connected in cascade until a voltage representing the variable itself is obtained. Connections are made from the outputs of the various integrators, either directly or through sign changers as the sign of the term in the equation dictates, to separate inputs to the summing amplifier whose output represents the highest order derivative. An independent forcing function and/or the output of another network representing the solution of another equation can be connected as additional inputs to this summing circuit. Constants of the summing or combining circuit, integrators, and sign changers must be adjusted to conform to the coefficients of the equation to be solved. The output of a computer connected as an integrator is equal to the negative of the integral of the input function divided by the product of the resistor and the capacitor. The gain of a sign changer or summing amplifier circuit is equal to the negative of the feedback resistor divided by the corresponding input resistor. These statements are true because the current flowing in the feedback impedance is equal to the sum of the currents flowing in all of the input impedances. In adjusting these constants, care must be taken to distribute constants and assign scales in such a manner that the average voltage input to each computer circuit is many times the potential of the pentode grid. There is also a maximum output voltage beyond which the circuits will not operate properly; this voltage limitation must also be considered in selecting the circuit constants. In the application of the computer amplifiers described in this report, no circuit should be required to provide more than 100 volts, either positive or negative.

NON-LINEAR EQUATIONS

A large number of non-linear equations may be solved by using the technique described under linear equations plus one or more multiplying circuits. A function to be the input of a computer circuit may be multiplied by another variable or some power of the function may be used as an input. A great many more may be solved by use of one or more devices for the insertion of arbitrary functions. A variable multiplied by an arbitrary function may be used as the input to a computer circuit etc.

DIFFERENTIATORS

As previously pointed out, when a computer amplifier is used as a differentiator it tends to be unstable requiring circuit modifications to prevent oscillation. Any method of preventing oscillation destroys the accuracy of computation. However, a true derivative may be obtained by a combination of circuits which avoid this difficulty.

Assume an equation:

$$y = x - \dot{x}$$

and its transposed form:

$$\dot{x} = x - y$$

Now if y is the output of one summing amplifier and $\dot{x}$ is the output of another, an integrator may be interconnected with these two circuits to provide derivative output. Since there is a sign change in each circuit, the inputs required for the first summing amplifier are $-x$ and $+\dot{x}$; for the second $-x$ and $+y$. These inputs may be provided by connecting the output of each circuit as an input to the other, integrating $\dot{x}$ to get $-x$, and the remaining $-x$ input is the function to be differentiated. Figure 9 shows this method schematically.

This same general scheme may be used to solve a higher order differential equations in which a given variable term is known but terms involving its derivatives are unknown. In this case, and in the case of most practical problems, the number of circuits required is not so great as Figure 9 would indicate. Most problems of this type would require only one amplifier in addition to the number required for a comparable problem in which the forcing function is the known quantity.

BOUNDARY CONDITIONS

The boundary conditions of a differential equation may be set in simply by determining the initial charge on the integrating capacitors. This may be done by switches or relays plus batteries for the integrators whose initial charge is not zero. An alternative method is to make the initial charge on all capacitors zero and to accommodate those functions which have initial values by a separate constant d-c input to those computer circuits whose inputs are connected to the integrator outputs involved.

PARTIAL DIFFERENTIAL EQUATIONS

Partial differential equations may be solved after they have been reduced to finite difference equations. The limitation is that time is the only independent variable.

ACCURACY

Accuracy of computation depends on several factors. Some are part of the computer amplifier design while others concern the manner in which the problem is set in and the type of problem. The following is a list of the more important factors which influence accuracy of computation:

1. Validity of equations simulated

2. Recording accuracy

3. Circuit voltage scale. In order that accurate results may be obtained, it is necessary in the case of each circuit that a voltage scale be assigned such that the average value of input voltage shall be much greater than the error in potential of the input grid. This result may be accomplished over a wide range of equation parameters by providing computing circuits capable of a large magnitude of output voltage. The circuits described provide ±100 volts.

4. Circuit resistor accuracy. The results can be no more accurate than the resistors used to set in the problem.

5. Circuit capacitor accuracy. Capacitors accurate in capacitance value and having low dielectric absorption properties must be used. Condensers suitable for this purpose are mica, polystyrene, and Sprague Vitamin Q.

6. Voltage regulation. Regulation of the direct-current supplies and the amplifier input tube heater supply is required. The units described are provided with electronically regulated direct-current supplies and resonant circuit type a-c heater voltage regulators.

7. Glow lamp stability. The glow lamps used must have

stable and consistent properties. Type OA3/VR75 gas regulator tubes were chosen because of their superior characteristics.*

CONCLUSIONS

It is concluded that NOTS is equipped to solve most problems involving linear differential equations by analog methods. In the near future many non-linear equations may be solved.

Vernon R. Briggs
VERNON R. BRIGGS

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*George M. Kirkpatric, Proc. I.R.E. Vol. 35, No. 5, May, 1947, P 485.

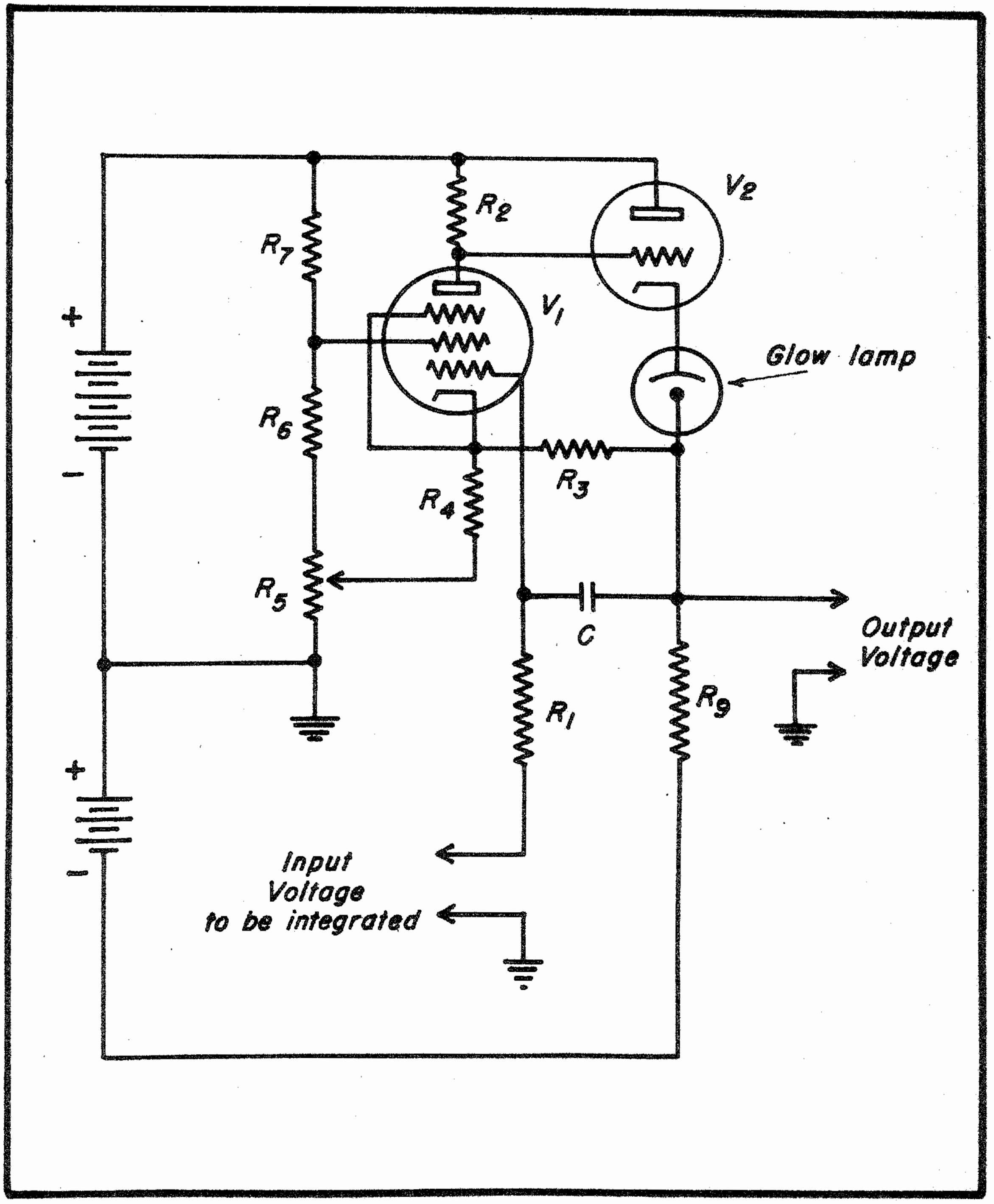

Figure 1. SCHEMATIC WIRING DIAGRAM OF THE BASIC INTEGRATING CIRCUIT.

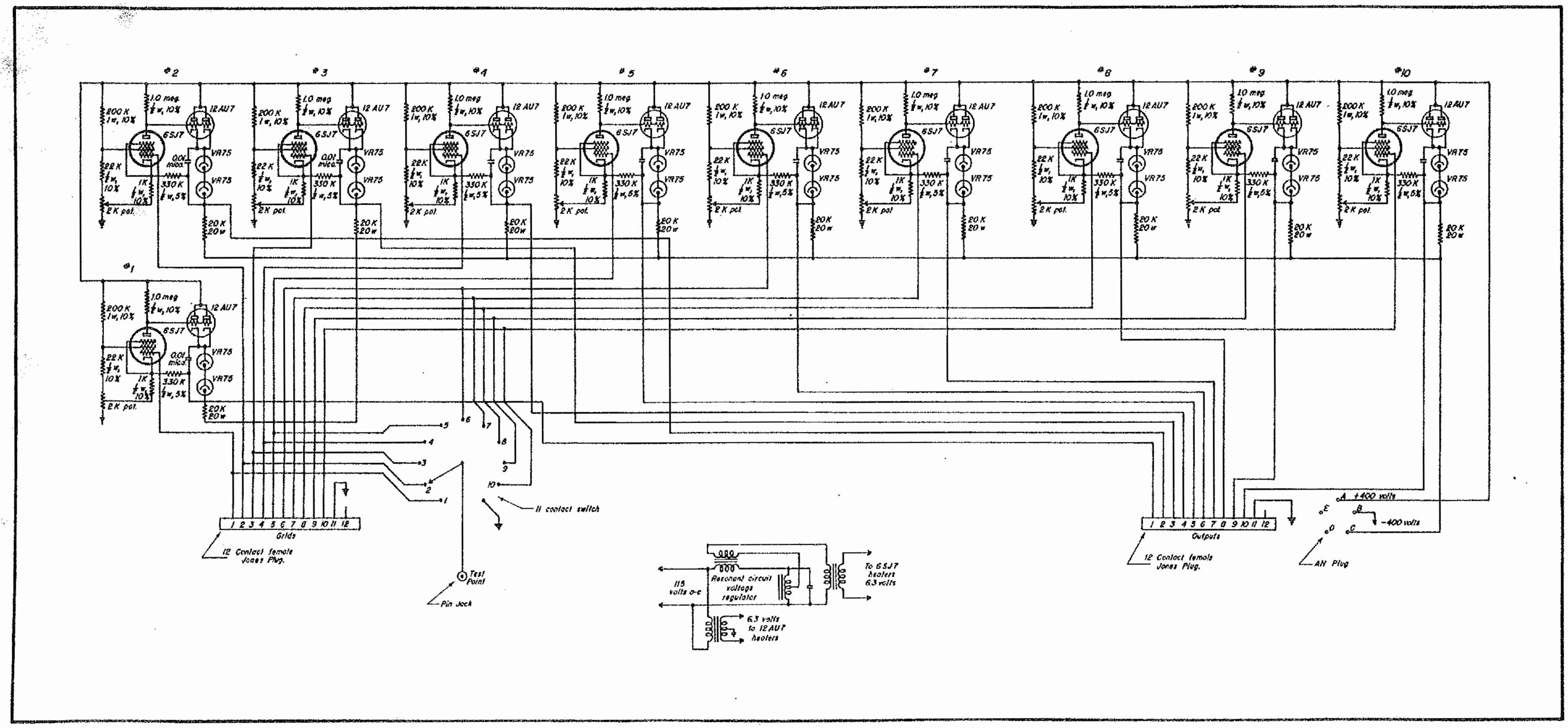

Figure 2. TEN-CHANNEL COMPUTER-AMPLIFIER.

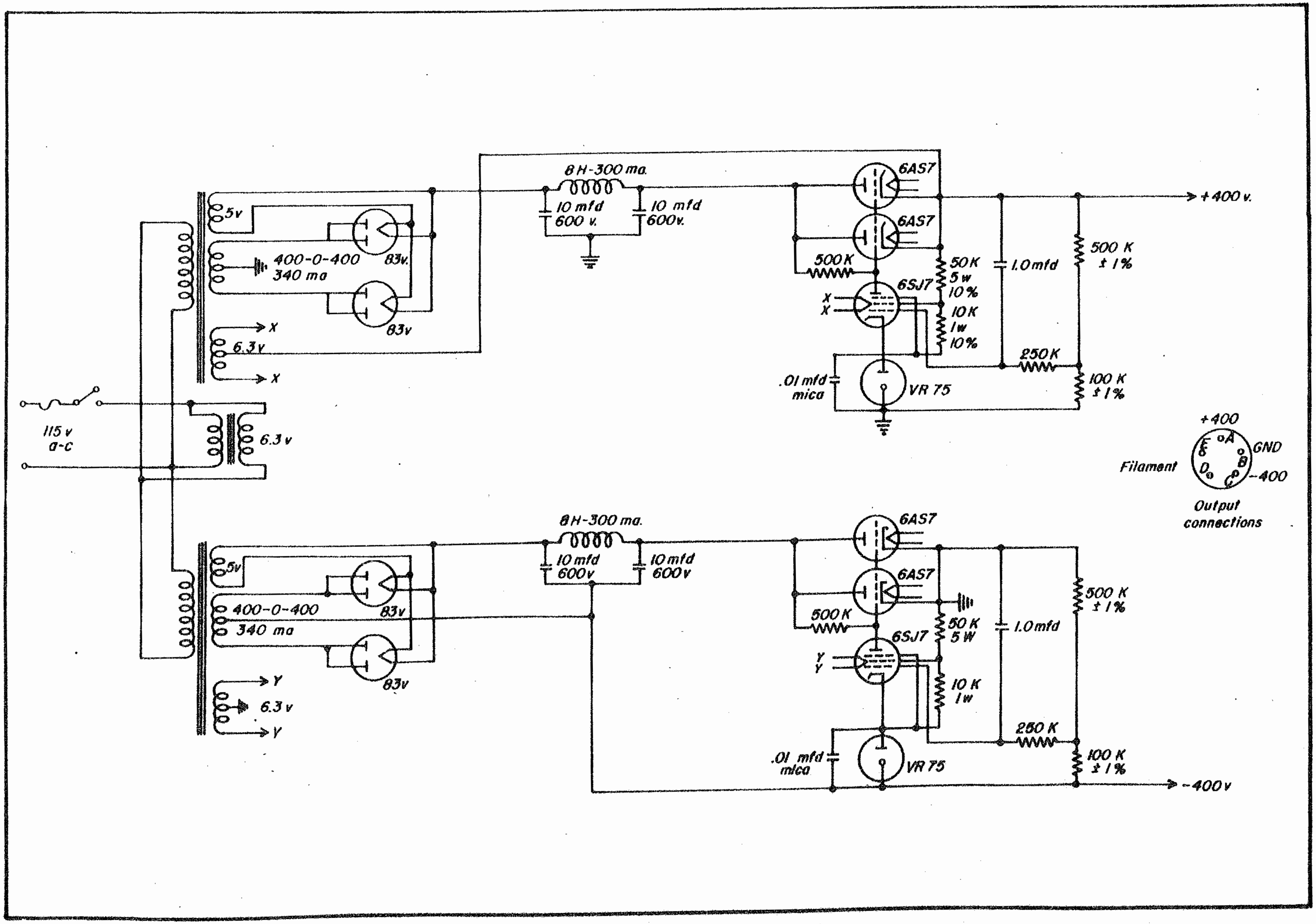

Figure 3. REGULATED POWER SUPPLY.

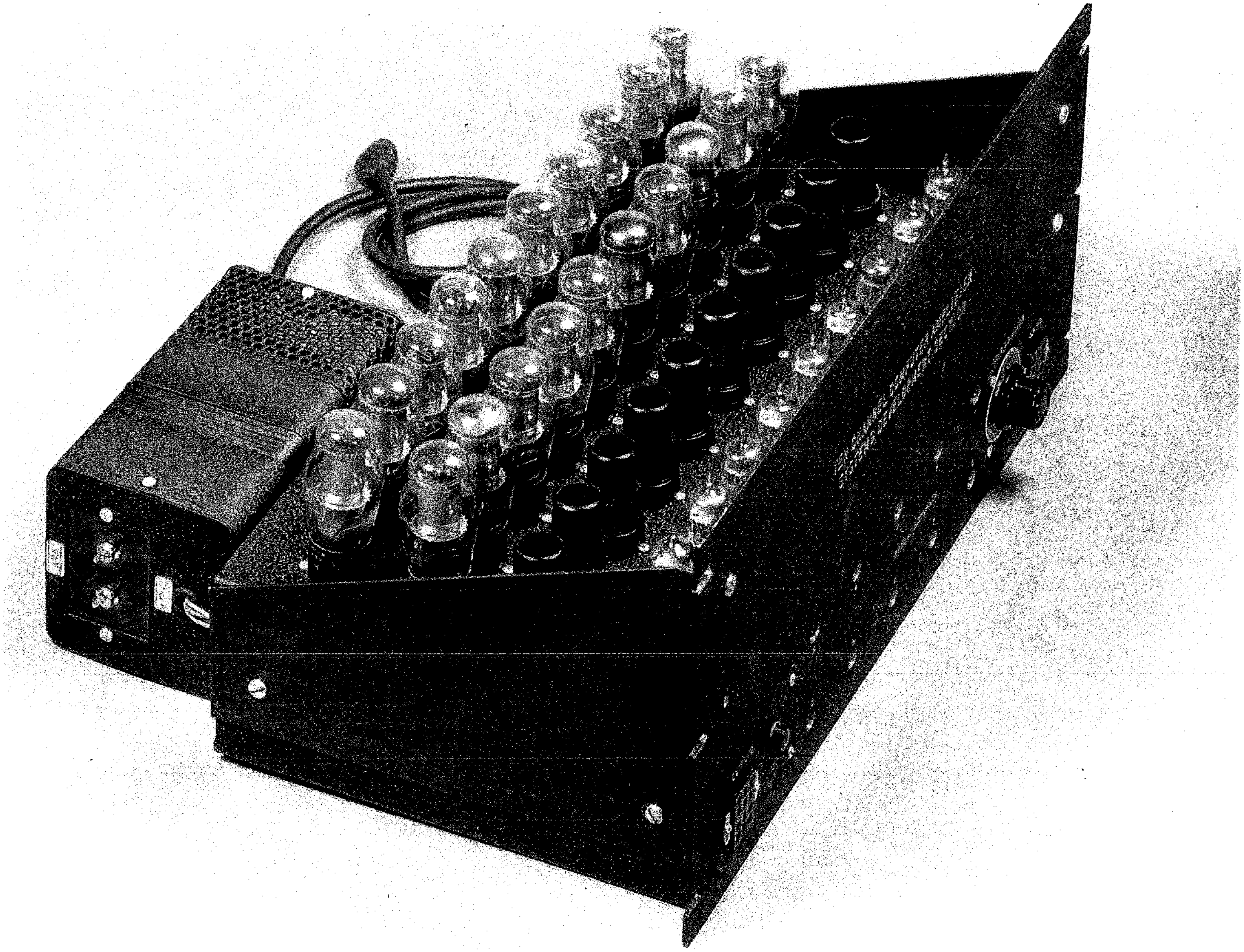

Figure 4. Ten-Channel Computer Amplifier
Unclassified NP/45 # 23773

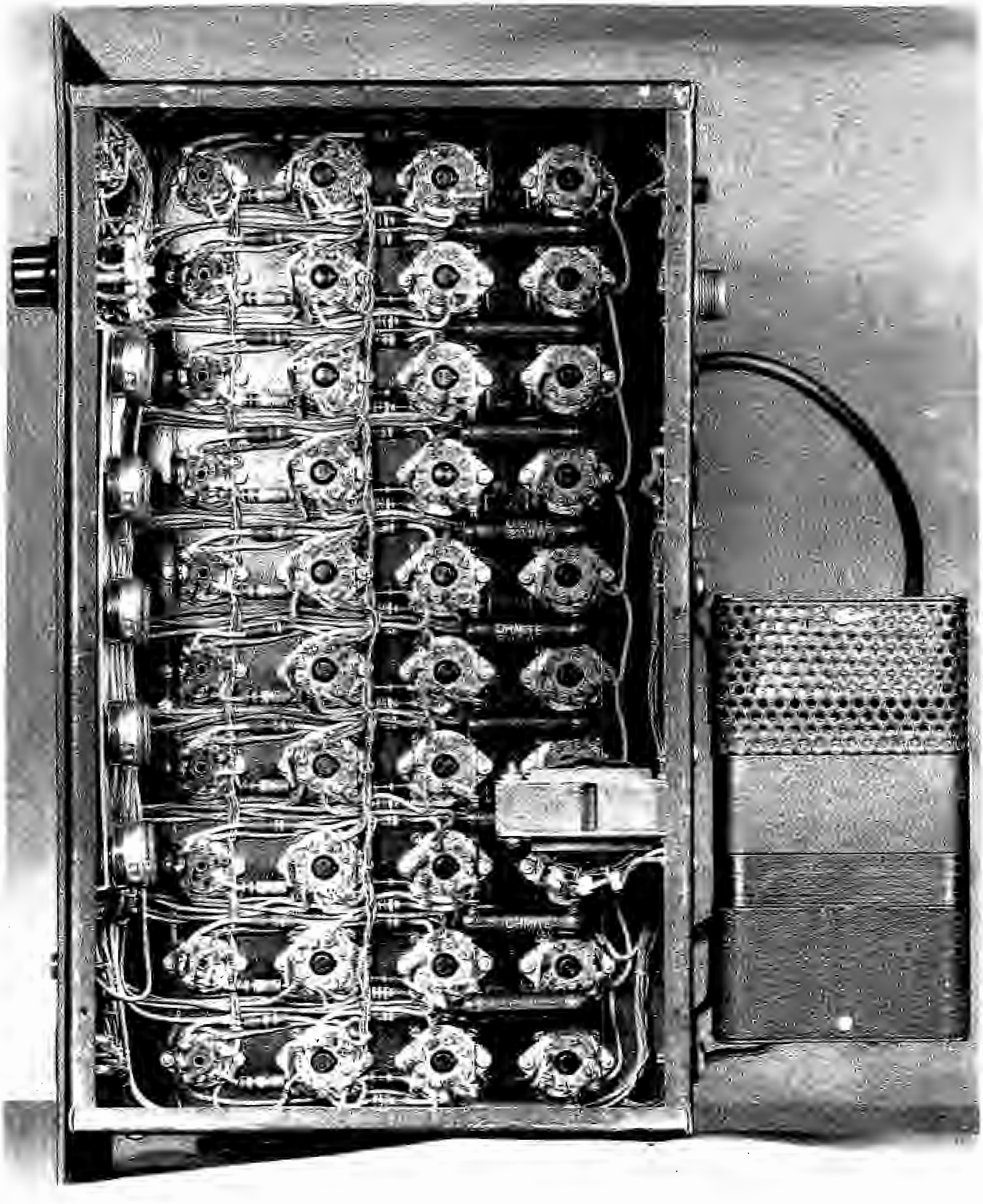

Figure 5. Bottom View of Ten-Channel Computer Amplifier
Unclassified NP/45 # 23774

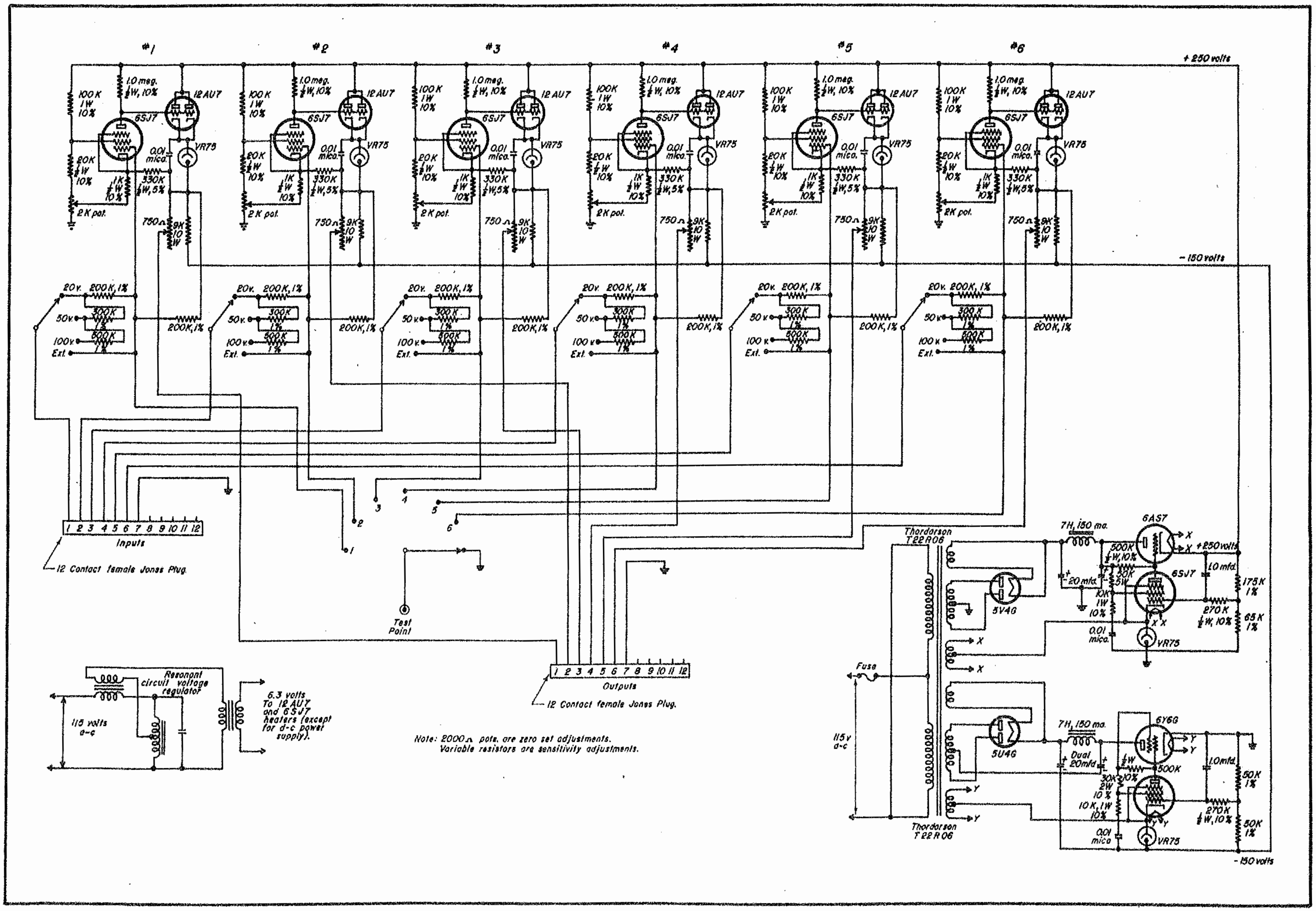

Figure 6. SIX-CHANNEL OSCILLOGRAPH AMPLIFIER-COMPUTER.

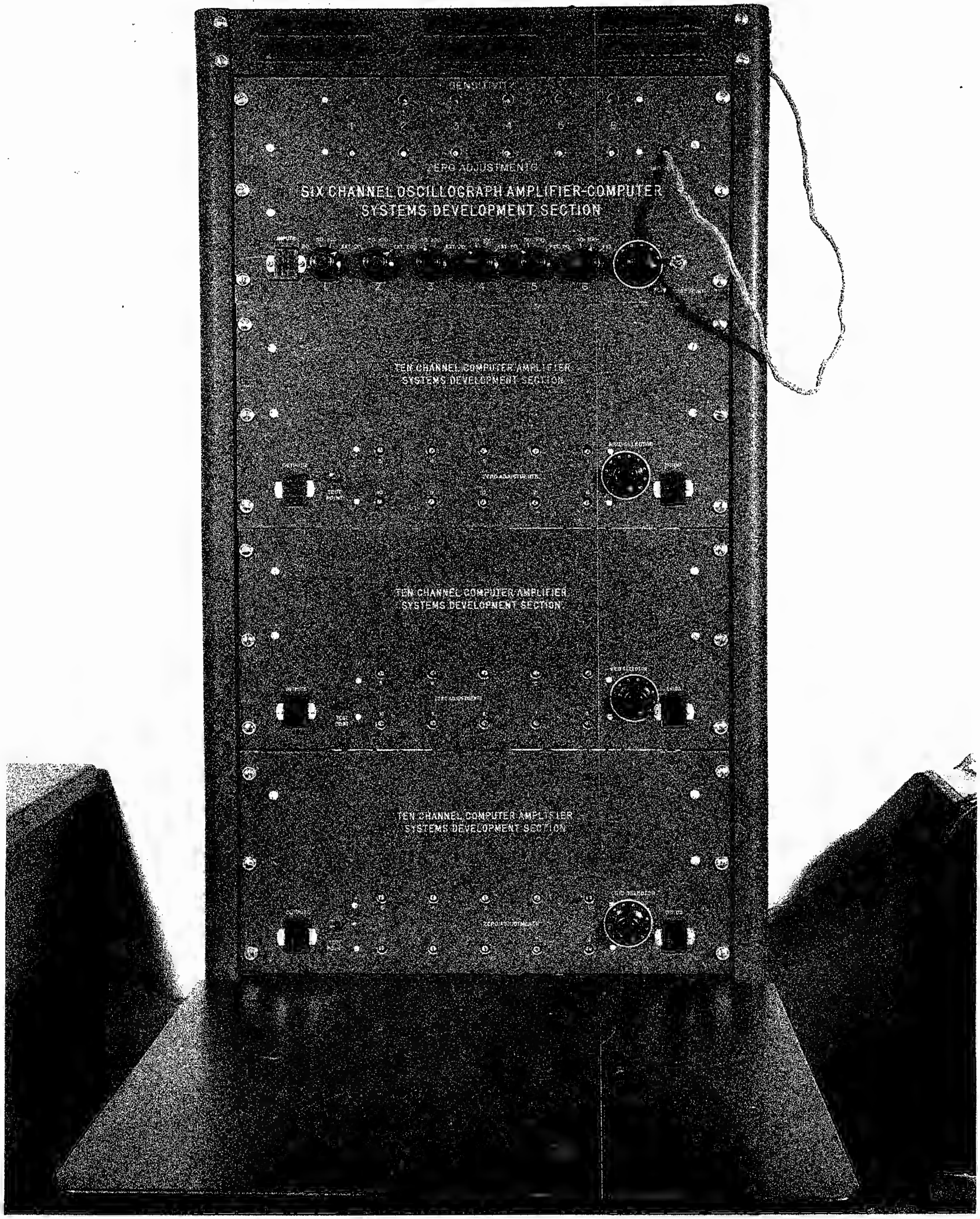
SENSITIVITY
ZERO ADJUSTMENTS
SIX CHANNEL OSCILLOGRAPH AMPLIFIER-COMPUTER
SYSTEMS DEVELOPMENT SECTION
TEN CHANNEL COMPUTER AMPLIFIER
SYSTEMS DEVELOPMENT SECTION
TEN CHANNEL COMPUTER AMPLIFIER
SYSTEMS DEVELOPMENT SECTION
TEN CHANNEL COMPUTER AMPLIFIER
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Figure 7. Rack Containing Three Ten-Channel Computer Amplifiers
Plus Six-Channel Oscillograph Amplifier-Computer

Unclassified NP/45 # 23772

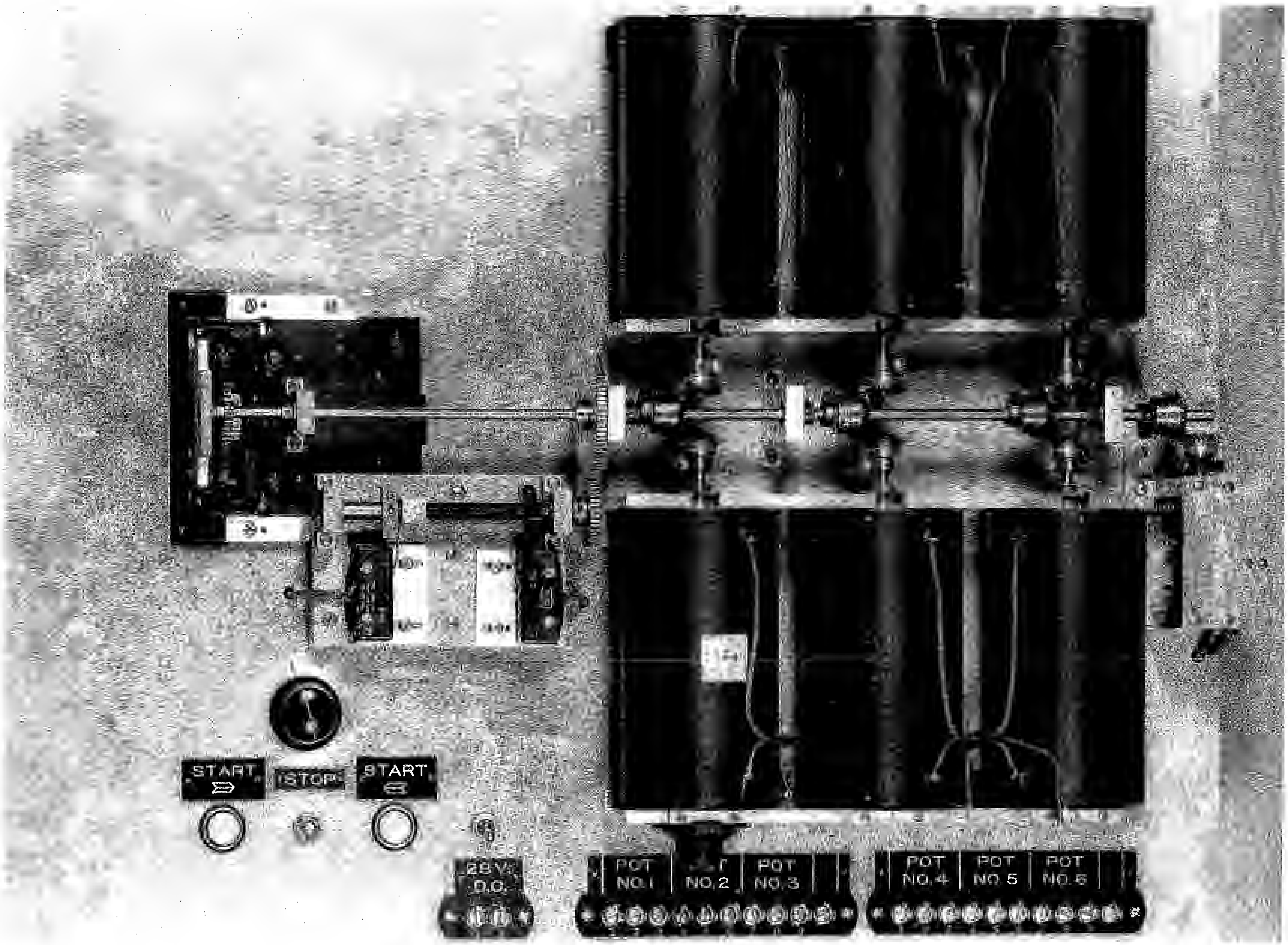
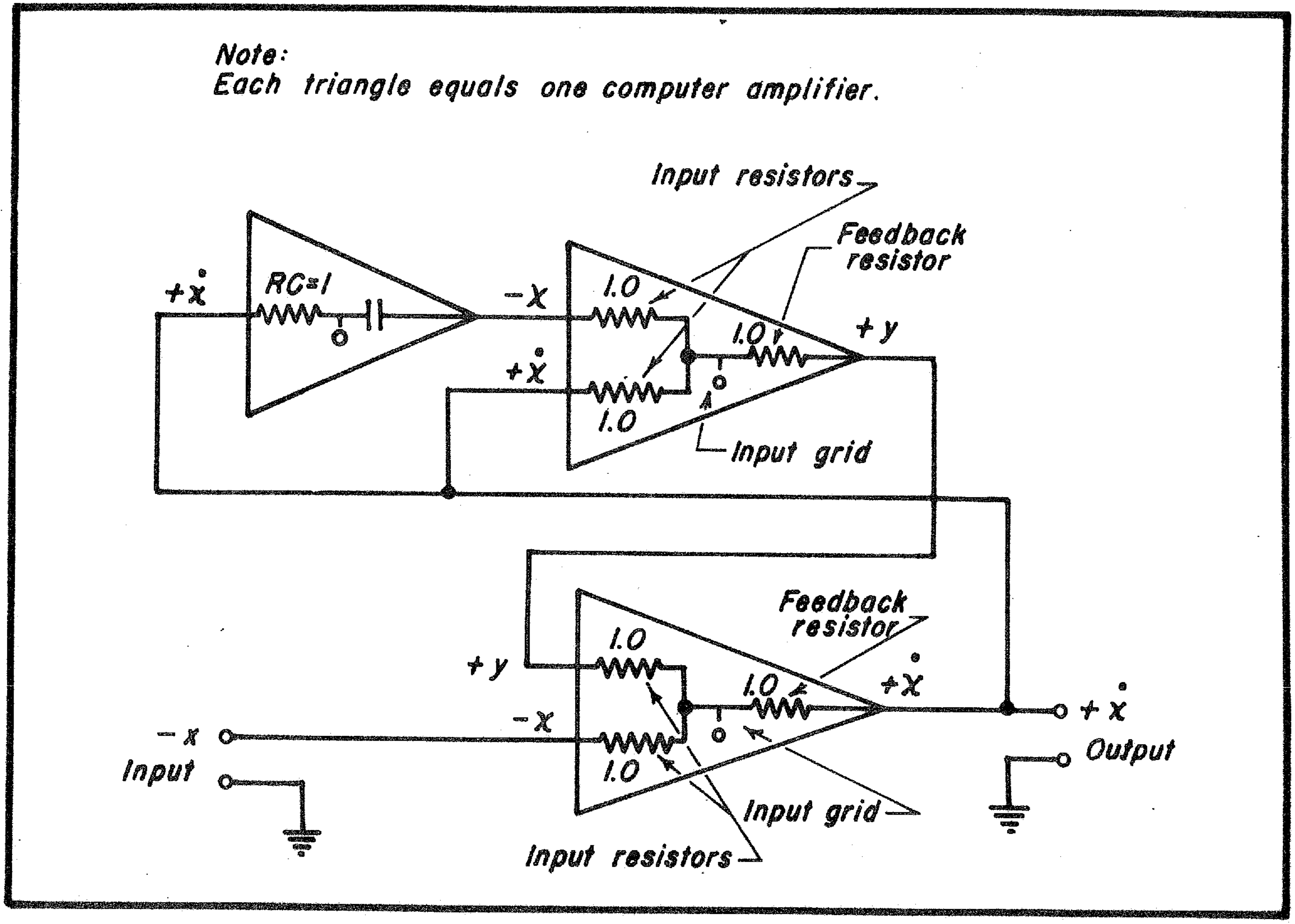


Figure 8. Six-Ganged Motor Driven Potentiometers
Unclassified NP/45 # 23771



Note:
Each triangle equals one computer amplifier.

Figure 9. SCHEMATIC WIRING DIAGRAM OF COMBINATION ACCOMPLISHING DIFFERENTIATION BUT AVOIDING DIFFICULTIES OF SIMPLE DIFFERENTIATOR.